

Problems

Ted Eisenberg, Section Editor

This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to <eisenbt@013.net>. Solutions to previously stated problems can be seen at <<http://www.ssma.org/publications>>.

*Solutions to the problems stated in this issue should be posted before
February 15, 2014*

- **5277:** *Proposed by Kenneth Korbin, New York, NY*

Find x and y if a triangle with sides $(2013, 2013, x)$ has the same area and the same perimeter as a triangle with sides $(2015, 2015, y)$.

- **5278:** *Proposed by Tom Moore, Bridgewater State University, Bridgewater, MA*

The triangular numbers $6 = (2)(3)$ and $10 = (2)(5)$ are each twice a prime number. Find all triangular numbers that are twice a prime.

- **5279:** *Proposed by D.M. Băţinetu–Giurgiu, “Matei Basarab” National College, Bucharest, Romania and Neculai Stanciu, “Geroge Emil Palade” General School, Buzu, Romania*

Let $f : \mathfrak{R}_+ \rightarrow \mathfrak{R}_+$ be a convex function on \mathfrak{R}_+ , where \mathfrak{R}_+ stands for the positive real numbers. Prove that

$$3(f^2(x) + f^2(y) + f^2(z)) - 9f^2\left(\frac{x+y+z}{3}\right) \geq (f(x) - f(y))^2 + (f(y) - f(z))^2 + (f(z) - f(x))^2.$$

- **5280:** *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*

Let $a \geq b \geq c$ be nonnegative real numbers. Prove that

$$\frac{1}{3} \left(\frac{(a+b)(c+a)}{2 + \sqrt{a+b}} + \frac{(c+a)(b+c)}{2 + \sqrt{c+a}} + \frac{(b+c)(a+b)}{2 + \sqrt{b+c}} \right) \leq \frac{(a+b)^2}{2 + \sqrt{b+c}}.$$

- **5281:** *Proposed by Arkady Alt, San Jose, CA*

For the sequence $\{a_n\}_{n \geq 1}$ defined recursively by $a_{n+1} = \frac{a_n}{1 + a_n^p}$ for $n \in \mathcal{N}$, $a_1 = a > 0$,

determine all positive real p for which the series $\sum_{n=1}^{\infty} a_n$ is convergent.

- **5282:** *Proposed by Ovidiu Furdui, Technical University of Cluj-Napoca, Cluj-Napoca, Romania*